RATIO & PROPORTIONALITY

Ratio is one of the best ways of comparing the size of different quantities.

You can compare the sizes of things directly but this turns out not to be very useful as is shown in this example.

Below is a photo of me with my grandson Felix. I would estimate that at the time this photo was taken, I was roughly 1m taller than Felix.

On the right. above is a picture of 2 London skyscrapers 40, Bank Street and 10 Upper Bank Street. 40 Bank Street is roughly 1m taller than 10 Upper Bank Street.

Strictly speaking, in absolute terms, the height difference is identical in the 2 situations, but clearly we would generally say that I am a great deal taller then Felix, whereas the 2 skyscrapers in London are almost exactly the same height.

The reason why this way of comparing size does not work is that it does not take account of the size of the objects to begin with.

This is where ratio comes in. It allows us to compare while taking account of the sizes of the objects (or people!)

NOTATION

Ratios are written using the following notation:

$$2:1$$

The two numbers to be compared are written either side of a colon.

If we return to our previous examples, the ratio of my height to Felix's height was **1.8 : 0.8**, because I am roughly 1.8m tall and Felix was roughly 0.8m.

If we now look at the skyscrapers, the ratio of 40 Bank St to 10 Upper Bank St is **152 : 151**, because the 2 buildings are 151m and 152m high respectively.

From this you can see, from a ratio point of view, that I am more than twice Felix's height in this photo, but the 2 buildings are almost the same height.

RECIPES

Now it turns out that ratios are seriously useful in everyday life. Of all the topics where the perennial student question "When will we ever need this in real life?" is entirely inappropriate, is with ratios. The most common place where we use them is in the kitchen; in recipes. Recipes in cookery book often assume that you are making meals for 4 people, but you may be cooking meals for fewer or more people, so the amount of ingredients required, needs to be adjusted accordingly.

Example 1: Let's look at a simple pancake recipe for 4 people.

Question: A recipe for pancakes, lists the following ingredients:

50g plain flour
1 medium egg
150ml semi-skimmed milk
0.5 tbsp cooking oil
1 pinch of salt

This will serve 4 people.
What quantities will be needed to serve 12 people?

Method: What must we multiply 4 by to get 12? (12 ÷ 4 = 3)
So 3 will be out multiplier.

Codes for ingredients: **F : E : M : O : S**

Quantities for 4 people: 50 : 1 : 150 : 0.5 : 1

Quantities for 12 people: 150: 3 :450 : 1.5 : 3

Answer: 150g plain flour
3 medium egg
450ml semi-skimmed milk
0.5 tbsp cooking oil
1 pinch of salt

Will be needed to feed 12 people.

It is important to realise that with ratios, we are always going to be multiplying or dividing, and **not** adding or subtracting, for the reasons outlined in the photo example above.

Simplifying Ratios

Ratios work a bit like fractions. They are not the same thing at all and we will examine the difference later on, but the "do the same thing to numerator and denominator" can be adapted to "do the same thing to all numbers in the ratios" and this will work just fine. This means that we can simplify (or complicate) ratios in much the same way we do with equivalent fractions. If this is new to you, please have a look at the companion document on **Fraction Arithmetic**.

Example 2: Cancelling ratios down to their simplest form.

Question: Tasnim is five years old and Ziad is twenty years old.
Write down the ratio of their ages in its simplest form.

Method:
T : Z
5 : 20
1 : 4 (divide both sides by 5)

Answer: 1 : 4

Consistent units

In the pancake recipe above, we muddled up lots of different units by way of measuring them in different ways. Milk was measured in ml; Eggs in units; Flour in grams; Oil in tablespoons; Salt in pinches. This is fine in the recipe scenario, but often, it is much more useful to make the units consistent if possible, as in the next example. This has the benefit of allowing you to use any units you choose, so long as you are consistent. In the next example we have minutes against hours, which is confusing. Writing both in the same units before simplifying gives us a ratio which can then be used with any units.

Example 3: Ratios with inconsistent units.

Question: Express 24 minutes to one hour as a ratio in its simplest form.

Method:
24 mins : 1 hour
24 mins : 60 mins
24 : 60 (now that the units are consistent, we can ignore them)
2 : 5 (divide both sides by 12)

Answer: 2 : 5
(The answer could be 2 hours : 5 hours or alternatively it could be 2 secs : 5 secs. The ratio notation will now work for you in any time units you choose.)

Exercise 1 – Manipulating Ratios

1. Simplify the following ratios into their simplest forms.
Note: Simplest form for a ratio, is where all numbers are natural (whole) numbers.

a) 8 : 10 **b)** 25 : 15 **c)** 36 : 54 **d)** $\frac{2}{3} : 6$ **e)** 1.8 : 4.2

2. Write the following ratios in the form 1 : *n*, where *n* is a positive number.
Note: *n* could be a whole number, a fraction or a decimal, if the decimal is not recurring.

a) 2 : 8 **b)** 5 : 3 **c)** 4 : 1 **d)** $\frac{5}{8} : \frac{3}{5}$ **e)** 1.4 : 3.8

3. Write the following ratios which all have inconsistent units, as unit-free, simplified ratios.

a) 2 hours : 30 mins **b)** 3.5kg : 400g **c)** 12 litres : 600 ml **d)** £5.60 : 70 p

4. The recipe for a salad dressing requires 10ml red wine vinegar, 50ml olive oil, 0.25tsp mustard, 0.5tsp sugar and one grind each of salt and pepper. It is suggested that these quantities will make enough salad dressing for 6 people. John is expecting 9 people for dinner, how much of each ingredient, will he require?

RATIOS & FRACTIONS

We talked a bit about the relationship between ratios and fractions in the preamble to eg2, but glossed over the differences between them. We will now look at how they are related and how they differ. Let us consider the ratio 3 : 7. For every 3 things I get, you get 7 things. Now the question we ask now is how many things is that? 3 for me, 7 for you, so 3 + 7 = 10 things altogether. So when we ask the question: What is the fraction of all the things that I get out of all of the things there are? I get 3, the total is 10, so the fraction is going to be $\frac{3}{10}$.

$\frac{3}{10}$ for me and $\frac{7}{10}$ for you. Now the fallacy people fall into here is the idea that the fraction could be $\frac{3}{7}$ simply by writing the ratio in fraction form: This is wrong, as it makes no logical sense and must be avoided by being prepared in advance to understand that we are talking about a number of things being split up, which, if it were to be reduced down to a single thing would be a ratio of fractions which will add up to 1, like this:

$\frac{3}{10} : \frac{7}{10}$ and $\frac{3}{10} + \frac{7}{10} = 1$

We will be using this idea that the sum of the parts of a ratio gives us the total number of **things** being divided up.

The ratio 4 : 9 can be thought of as 4 for me and 9 for you, so 13 altogether, hence the ratio can be written as $\frac{4}{13} : \frac{9}{13}$.

Additionally, we can see that the sum of the fractions in this new ratio, will be $\frac{4}{13} + \frac{9}{13} = \frac{13}{13} = 1$.

This is something which always happens. If you write the ratio in terms of fractions like this, the sum of the fractions in the ratio will always be 1.

Example 4: Ratios and fractions.

Question: The pink paint that Robert used to paint his bathroom was made up using 1 litres of white paint mixed with $\frac{1}{4}$ litres of red paint.

a) Write down the ratio of white paint to red in its simplest form.
b) What fraction of the pink paint is red?
c) What fraction of the pink paint is white?

Method: White : Red

1 : $\frac{1}{4}$

a) 4 : 1 (multiply by 4)
b) There are 4 + 1 = 5 parts altogether, so that gives a denominator of 5
So the fraction of the pink which is red is $\frac{1}{5}$.
c) If it's not red, it is white so $\frac{4}{5}$ of the paint is white.

Answer:

a) 4 : 1
b) $\frac{1}{5}$
c) $\frac{4}{5}$

Example 5: Ratios and fractions.

Question: A garden is divided up into 5 parts lawn and 3 parts shrubs.
What fraction of the lawn is covered by
a) lawn,
b) shrubs?

Method: Lawn : Shrub
5 : 3

a) 5 + 3 = 8 so there are 8 parts altogether
so $\frac{5}{8}$ is lawn.

b) and $\frac{3}{8}$ is shrub.

Answer:
a) $\frac{5}{8}$
b) $\frac{3}{8}$

Exercise 2 – Ratio & Fractions

1. Dev and Shona share a pizza in the ratio 3 : 5 and they eat all of it.

a) What fraction of the pizza did Dev eat?
b) What fraction did Shona eat?

2. A length of copper pipe is to be cut into 2 pieces in the ratio 3 : 7. What fraction of the original length is the shorter of the two pieces?

3. Beth gets $\frac{3}{7}$ of a bag of sweets. Her cousin, Liz gets the rest. Work out the ratio of sweets each receives. Write your answer in the form Beth : Liz.

4. A soccer team play 16 matches and win $\frac{3}{4}$ of them. They play another p matches and win them all.
PS The ratio of wins to losses is then 4 : 1. Calculate the value of p.

5. Alex, Brian & Chinua share some money. The ratio of Alex's share to Brian's share is 1 : 2. The ratio of Brian's share to Chinua's share is also 1 : 2. What is the ratio of Alex's share to Chinua's
PS share.

6. Three sisters, Amy, Beth and Charlotte share a bar of chocolate in the the ratio of their ages. Amy gets half the bar of chocolate, Beth gets $\frac{3}{5}$ of what is left and Charlotte gets the remaining portion.
PS

a) Work out the ratio of what each receives in the form A : B : C, where A, B & C are whole numbers.
b) Charlotte is 8 years old. How old is Amy?

Dividing a quantity in a given ratio

We can use the ideas above to do much of the work when we want to divide up a quantity according to a given ratio.

The method for dividing a quantity in a given ratio is simple and requires that we think about the numbers in a ratio as **the value of parts**. In the ratio 4 : 3, we have 4 parts and 3 parts so 7 parts altogether. Now the value of a single **part** can be found by dividing the quantity by the number of parts. Once we have the value of one part, we can then mutiply each side of the ratio by the value of one part. This can be seen clearly in the next example.

Example 6: Dividing up a quantity in a given ratio.

Question: Divide £28 in the ratio 4 : 3.

Method: The ratio is 4 : 3, so there are 4 + 3 = 7 parts in all.
We can calculate the value of 1 part by dividing the quantity (£28) by the number of parts (7). This gives that 1 part is worth 28 ÷ 7 = £4

Ratio is 4 : 3
Each part is worth £4 so we can multiply both numbers by 4.

New ratio is 4 × 4 = 16 : 3 × 4 = 12
16 : 12. (Note that 16 + 12 = 28, which is what we expect.)

Answer: £28 divides up into £16 : £12

Exercise 3

1. Share 30kg in the ratio 1 : 2.

2. Share £56 in the ratio 3 : 4.

3. Share £480 in the ratio 1 : 2 : 5.

4. Andy and Annie share out sweets between them in the ratio of their ages: 6 and 4 years old respectively. If they have 50 sweets to share out, how many do each receive?

5. Richard intends to give Jimi and Felix £84. He wants to divide the money so that Jimi gets twice as much as Felix. How much will they each receive?

6. Three children share out 260 marbles in the ratio 2 : 3 : 5. How many marbles does the one who get the largest number receive?

7. Find the smallest share when 260 g is divided up in the ratio 4 : 2 : 7.

8. Share £252 so that A has half as much as B, who has 4 times as much as C.

Answers

Exercise 1

1. **a)** 4 : 5 **b)** 5 : 3 **c)** 2 : 3 **d)** 1 : 9 **e)** 3 : 7

2. **a)** 1 : 4 **b)** 1 : 0.6 **c)** 1 : 0.25 **d)** $1 : \frac{24}{25}$ **e)** $1 : \frac{19}{7}$

3. **a)** 4 : 1 **b)** 35 : 4 **c)** 20 : 1 **d)** 8 : 1

4.

Vinegar	Oil	Mustard	Sugar	Salt	Pepper
15 ml	75ml	0.375 tsp	0.75 tsp	1.5 grinds	1.5 grinds

Exercise 2

1. **a)** $\frac{3}{8}$ **b)** $\frac{5}{8}$

2. $\frac{3}{10}$

3. 3 : 4

4. $p = 4$

5. 1 : 4

6. **a)** 5 : 3 : 2 **b)** Amy is 20

Exercise 3

1. 10 kg : 20 kg **2.** £24 : £32 **3.** £60 : £120 : £300 **4.** 30 : 20 **5.** Jim gets £56, Felix gets £28

6. 130 **7.** 40 **8.** A has £72, B has £144, C has £36.