

TEACHING PERCENTAGES

Q: What is a percentage?

A: A fraction out of 100.

Q: How does it differ from a fraction?

A: Just in its denominator, which is always 100.

Q: What does the word **of** mean in mathematics?

A: **of** means multiply. Always!

Q: Why are percentage questions hard?

A: Percentage questions are difficult, but they are also worth persevering with them because they are so useful. They can be used in so many contexts that it is very easy to muddle up what kind of question it is that you are trying to answer. Interpretation of the question, then, is the most important first step to working with percentages.

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CONVERTING TO AND FROM PERCENTAGES

It is important to be able to be able to write fractions in a variety of forms and to be able to convert easily between them.

Example 1: Converting 1 significant figure decimal fractions to percentages by multiplying them by 100.

Question: Convert 0.9 to a percentage.

Method: $0.9 \times 100 = 90$

Answer: $0.9 = 90\%$

Example 2: Converting 2 significant figure decimal numbers to percentages.

Question: Convert 0.46 to a percentage.

Method: $0.46 \times 100 = 46$

Answer: $0.46 = 46\%$

Example 3: Converting decimal numbers greater than 1 to percentages.

Question: Convert 1.03 to a percentage.

Method: $1.03 \times 100 = 103$

Answer: $1.03 = 103\%$

Example 4: Converting common fractions which produce terminating decimals to percentages.

Question: Convert $\frac{3}{5}$ to a percentage.

Method: $\frac{3}{5} \times 100 = 0.6 \times 100 = 60$

Answer: 60%

Example 5: Converting fractions which produce recurring decimals to percentages.

Question: Convert $\frac{3}{7}$ to a percentage.

Method: $\frac{3}{7} = 0.42857\dot{1}$
 $0.42857\dot{1} \times 100 = 42.8571\%$

Answer: 42.9% (1 d.p.)

EXERCISE 1

- Convert the following decimal numbers to percentages.
 - 0.4
 - 0.3
 - 0.8
 - 0.2
 - 0.1
 - 0.5
- Convert the following decimal numbers to percentages.
 - 0.14
 - 0.78
 - 0.03
 - 0.99
 - 0.05
 - 0.999
 - 0.45
 - 0.125
- Convert the following decimal numbers to percentages.
 - 1.2
 - 3.2
 - 1.05
 - 2.54
 - 2.3
 - 1.03
 - 2.5
 - 1.175
- Convert the following fractions to percentages.
 - $\frac{1}{2}$
 - $\frac{3}{10}$
 - $\frac{1}{4}$
 - $\frac{7}{10}$
 - $\frac{3}{4}$
 - $\frac{9}{10}$
 - $\frac{1}{10}$
 - $\frac{2}{5}$
- Convert these fraction to percentages, giving your answers to 1 decimal place.

a) $\frac{2}{3}$

b) $\frac{1}{8}$

c) $\frac{2}{7}$

d) $\frac{2}{9}$

e) $\frac{4}{9}$

f) $\frac{5}{6}$

g) $\frac{5}{7}$

h) $\frac{3}{11}$

6. Copy the table below into your exercise book and fill in the cells in the table below.

Fraction	Decimal	Percentage
$\frac{1}{2}$		
—	0.4	
—		4%
—	0.35	
$\frac{3}{8}$		
—	0.03	

FINDING PERCENTAGES OF A QUANTITY

If units are given in the question, remember to give the units for your answers.

If you round an answer, be sure to state how you rounded the number (eg 3 s.f. or 2 d.p. etc.)

Finding a percentage of some quantity is just like finding a fraction of the quantity. The only distinction is that we will always use the same denominator, 100.

Example 6: Finding a percentage of a quantity. Remember, in mathematics, **of** always means **multiply**.

Question: Find 35% of 84

Method: Convert the percentage to a decimal and then multiply.

$$35\% = 0.35$$

$$0.35 \times 84 = 29.4$$

Answer: 29.4

Example 7: Simple reverse percentage questions. Working backwards in time.

Question: 24% of a number is 34.56. What is the number?

Method: **Method 1: Ratios**

$$24\% : 34.56$$

$$1\% : 1.44$$

$$100\% : 144$$

Answer: 144

Method 2: Algebraic

$$24\% \text{ of } x \text{ is } 34.56$$

$$0.24x = 34.56$$

$$x = 34.56 \div 0.24 = 144$$

EXERCISE 2

- Work out the following
 - 10% of £80
 - 20% of 100m
 - 30% of 60cm
 - 40% of 130kg
 - 80% of £300
 - 100% of 65g
- Calculate
 - 12% of 45
 - 52% of 38
 - 66% of 66
 - 56% of 12
 - 98% of 144
 - 6% of 88
- Bill spends 40% of his pocket money on snacks. He receives £5.00 pocket money each week. How much money does he spend on snacks each week?
- Dylan has a box of chocolates containing 30 chocolates. He eats 20% of the chocolates. How many chocolates does he eat?
- Edith has 600 songs on her phone. 68% of them are recordings of songs by Franz Schubert. How many of the songs are by Schubert?
- Freddie is an insurance sales person. He receives a 5% commission on each policy he sells. In one week he sells £15 000 worth of policies. How much commission will he receive for that week?
- In a week in January, 18% of the pupils at a secondary school were absent. There are 1204 pupils in the school. How many students were present?
- Eve spends 30% of each evening between 6pm and 9pm doing homework. How long (in minutes) does she spend on her homework?
- In a bag containing 70 sweets, 30% are red. How many sweets are red?
- A school has 1250 pupils. 52% of the pupils are girls. How many boys are there in the school?
- In a jar of drawing pins, 15% of the pins are known to be yellow. If there are 720 pins in the jar, how many of them are not yellow?
- Roughly 12% of people are left-handed. In a randomly selected group of 400 people, how many would you expect to be right handed?
- George says that $a\%$ of b items is exactly the same as $b\%$ of a items. Is he correct? You must show all of your working.
- Calculate 83% of 35% of £12 000.
- Calculate 30% of 30% of 30% of $\frac{5}{27}$.
- 10% of a number is 6. What is the number I started with?
- 15% of a number is 9. What is the number I started with?
- Jimi spent 20% of his pocket money on sweets. He spent £1.30 on sweets. How much pocket money does Jimi receive?
- Felix has a bag of marbles. He give 30% of his marbles to Alexander. He gives Alexander 15 marbles. How many marbles did Felix have to begin with?
- Samira received money for her birthday from several relatives. She intends to spend £35.00 on comic books. This is 40% of the money she received. How much money did she get for her birthday?
- 55% of the students in a school are girls. There are 330 girls in the school. How many students are there in the school altogether?
- Alice did a maths test and scored 27 marks. She scored 45% of the available marks. What was the total number of marks available on the test?

23. 54% of the books in a library are non-fiction. There are 13 284 non-fiction volumes in the library. How many books does the library have altogether?

INCREASING A QUANTITY BY A PERCENTAGE

We regularly need to be able to find the result of increasing a number by a percentage.

In other words, we need to find the increase and then add it onto the original.

Using multipliers we can do this in a single step.

Example 7: Use a multiplier to solve problems where numbers are increased by a percentage.

Question: Increase £250 by 48%.

Method: $\text{Multiplier} = \frac{100 + p}{100} = \frac{100 + 48}{100} = 1.48$

$$250 \times 1.48$$

Answer: £370

EXERCISE 3

- Write down the multiplier used to increase a quantity by the following percentages:

a) 40%	b) 53%	c) 6%
d) 17.5%	e) 200%	f) 350%
g) 100%	h) 1%	i) 0.3%
- | | | |
|-------------------------|-------------------------|------------------------|
| a) Increase 100 by 1%. | b) Increase 90 by 10%. | c) Increase 14 by 5%. |
| d) Increase 18 by 50%. | e) Increase 45 by 15%. | f) Increase 60 by 12%. |
| g) Increase 152 by 12%. | h) Increase 144 by 25%. | i) Increase 82 by 15%. |
- | | | |
|---------------------------|----------------------------|----------------------------|
| a) Increase £6 by 10%. | b) Increase 5 kg by 30%. | c) Increase 18 cm by 45%. |
| d) Increase £65 by 18%. | e) Increase 42 l by 5%. | f) Increase £48 by 350%. |
| g) Increase 134 m by 48%. | h) Increase 304 cm by 12%. | i) Increase 55 mm by 112%. |
- A car was advertised for £9500 but is increased by 15%. What is the new price?
- Trevor takes his cat to the vet. The last time the cat was weighed it was 4kg and this has increased by 5%. What does Trevor's cat weigh now?
- A restaurant adds a service charge of 15% to every diner's bill. How much will a diner pay altogether if their meals came to £64 before the service charge was added?
- A population of seals on an island has been increasing at 6% per year. At the beginning of a year, the population was 850 seals. How many seals are there on the island one year later?
- A holiday package was advertised at £2500. The price goes up by 8%. What is the new cost of the package holiday?
- Sales tax (VAT) is added to all goods at 20%. What is the full price of an item priced at £58 before VAT is added?
- Bilal's energy bill in November came to £215. In December his energy company raises prices by 4%. Assuming that he uses the same amount of energy, what will his December bill come to?

11. The price of carrots goes up by 25% due to a shortage. If 1kg of carrots cost £1.12 before the rise, what will the new price be?
12. Deana gets a pay rise of 3%. Her monthly salary before the rise was £1780. How much does she earn after the pay rise?
13. In a clothes shop, all items are subject to a price rise of 4.5%. What will the new price be of a blouse which previously was priced at £34.99? Give your answer to the nearest penny.
14. Harry worked 35 hours in one week. The following week his working hours increased by 15%. How many hours did Harry work in the second week? (Give your answer in hours and minutes, to the nearest minute.)
15. The average rainfall in Birmingham in August 2016 was 50mm. The rainfall in September was 20% greater than in August. What was the average rainfall in Birmingham in September?

DECREASING A QUANTITY BY A PERCENTAGE

Likewise we often need to be able to find the result of decreasing a number by a percentage.

In other words, we need to find the decrease and then subtract it from the original.

Once again, using multipliers we can do this in a single step.

Example 8: Use a multiplier to solve problems where numbers are decreased by a percentage.

Question: Decrease 114cm by 42%.

Method: Multiplier = $\frac{100 - p}{100} = \frac{100 - 42}{100} = 0.58$

$$114 \times 0.58$$

Answer: 66.12 cm

EXERCISE 4

- Calculate the multiplier used to decrease a quantity by the following percentages:

a) 10%	b) 20%	c) 1%
d) 34%	e) 28%	f) 42%
g) 6%	h) 8%	i) 2.5%
- | | | |
|-------------------------|-------------------------|--------------------------|
| a) Decrease 100 by 10%. | b) Decrease 400 by 20%. | c) Decrease 1600 by 40%. |
| d) Decrease 324 by 18%. | e) Decrease 289 by 17%. | f) Decrease 256 by 16%. |
| g) Decrease 144 by 12%. | h) Decrease 512 by 8%. | i) Decrease 27 by 3%. |
- | | | |
|----------------------------|----------------------------|----------------------------|
| a) Decrease £28 by 40%. | b) Decrease £345 by 24%. | c) Decrease £879 by 21%. |
| d) Decrease 766 cm by 11%. | e) Decrease 45 m by 12%. | f) Decrease 578 m by 45%. |
| g) Decrease 555 m by 69%. | h) Decrease 654 mm by 72%. | i) Decrease 321 cm by 92%. |
- In a sale, the price of all vehicles in a showroom are to be reduced by 12%. How much would you pay for a car originally priced at £12 500?
- A carpenter needs to shorten a piece of wood of length 1320 mm by 13%. How long will the longer piece of wood be after the excess has been cut off? Give your answer to the nearest mm.
- A shirt priced originally at £56 is now in a sale marked as "15% off". How much would you pay for the shirt in the sale?
- Between January and February of 2016 the average rainfall in a British seaside resort, fell from 70mm by 29%. What was the average rainfall in February?
- William went on a diet at the beginning of January. Before starting the diet, William's mass was 89kg. By the end of February his mass had reduced by 5.6%. What was his mass at the end of February? (Give your answer to the nearest kg.)
- Computer equipment loses roughly 25% of its value each year. John bought a laptop for £360 last year. How much will it be worth one year later?
- CD sales declined sharply in the 2010s. Between 2015 and 2016, sales were down by 13.6%. If 100 000 000 CDs were sold in 2015, roughly how many were sold in 2016? Give your answer to the nearest hundred thousand.
- An investment of £2500 that Gillian made in January 2014 made a loss of 12% in the first year.
 - How much was the investment worth in January 2015?

In 2016, the investment did better and increased by 12%.

- b) How much was Gillian's investment worth in January 2017?
 c) Comment on the result to part b.
12. A shop increased all of its prices by 10% in March, only to reduce them by 10% in April. An employee says that the prices will go back to where they were. Was the employee correct? Give reasons for your answer.
13. Over the course of three years, the cost of a computer component fell in the first year by 14%, in the second year by 16% and in the third year by 15%. By what percentage did the price fall over the three years? Give your answer to 3 significant figures.

REPEATED INCREASES AND DECREASES OVER TIME.

Increases and decreases tend to recur over time and while it might appear complicated, the multiplier method can be simply adapted to take account of changes which occur every week, month or year. We know how to increase or decrease a quantity once so we will now look at how we can use the same methods to deal with problems where the amount changes by a fixed percentage over time. Examples of repeated increase include compound interest and repeated decrease can be described in terms of depreciation or the loss of value of an item, like a car, over time.

In this exercise we will examine examples of types like eg 10 below.

Example 10: This example deals with repeated increase using compound interest.

Question: £200 is invested at an annual interest rate of 5%.

What is the value of the investment after 1 year?

What is the value of the investment after 2 years?

What is the value of the investment after 10 years?

After how long will the investment reach a value of £500?

Method: A 5% increase will give a multiplier of 1.05.

So after one time period (in this case 1 year) the investment will be worth $200 \times 1.05 = 210$

A 5% increase over 2 years means that we must do the multiplication twice so we now have

$$200 \times 1.05 \times 1.05 = 200 \times 1.05^2 = 220.50$$

Clearly it would be daft to keep using this method for bigger numbers and we should jump straight to the power method so after 10 years (in general, 10 time periods) we will have $200 \times 1.05^{10} = 325.78$

In general then, we can say that any investment increased over time can be calculated using the general formula

$$\text{Value after } p \text{ time periods} = \text{principal} \times (\text{multiplier})^p$$

$$200 \times 1.05^p > 500$$

$$1.05^p > 2.5$$

Now use trial and improvement.

$$1.05^{18} = 2.41 \text{ (3 s.f.)}$$

$$1.05^{19} = 2.53 \text{ (3 s.f.)}$$

So the investment will reach £500 after 19 years.

Answers: £210

£220.50

\$325.78

19 years

Example 11: Repeated decrease works in exactly the same as repeated increase. The only difference is that the multiplier will be between 0 and 1.

Question: A car loses its value each year at a rate of 23% per year in its first 6 years. Colin bought a car for £17 500 in January 2020.

How much will the car be worth after 1 year?

How much will the car be worth after 4 years?

After how long will the car be worth less than £5 000?

Method: Multiplier for a 23% decrease is given by $1 - 0.23 = 0.77$

$$17500 \times 0.77 = 13\,475$$

$$17500 \times 0.77^4 = 6151.78$$

$$17500 \times 0.77^p < 5000$$

$$17500 \times 0.77^5 = 4736.87$$

Answers: £13 475

£6151.78

the car will be worth less than £5000 after 5 years.

EXERCISE 5

- Work out the value of an investment with a principal of £350, at the end of the following time periods, when the interest rate is 4% per year.
 - After 1 year.
 - After 3 years.
 - After 20 years.
- A grey seal pup, born with a body mass of 15kg, increases its body weight by 14% per day while it is being weaned. What will the seal pup weight after
 - 1 day
 - 2 days
 - 1 week
- A cypress hedge increases in height by 17% per year for its first 20 years. Jill bought a bunch of cypresses as a border for her garden. They were all 1.5m tall when purchased.
 - How tall would she expect the hedge to be after 5 years?
 - How long will it take for the hedge to reach a height of 9m?
- Scottish harbour seal populations have been declining at an average rate of 9% per year between 2000 and 2018. In 2000, Orkney's harbour seal population was 8500.
 - What was the population of harbour seals in Orkney by 2018?
 - If the decline continues at the same rate, what will the population of harbour seals be in 2023?
- The value of a washing machine when new is £560. The annual rate at which its value falls is 42%. What is the value of the machine after 4 years?

6. A car loses 35% of its value each year. How much will my car be worth after 7 years, if I originally paid £16 500?
7. Luis buys a new car for £28 400.
The value will depreciate by 20% in the first year and by 10% in each subsequent year. Find the value of the car after 3 years.
8. £3000 is invested at an interest rate of 6% for 7 years. Work out the total interest earned over the course of the 7 year period.
(Hint: you are asked for the interest only here, not the final value of the investment!)

EXPRESSING ONE QUANTITY AS A PERCENTAGE OF ANOTHER

The amount by which something changes is always dependent on the size of the object itself. For example if an elephant grows taller by 1mm, we would consider that to be negligible (unimportantly small) but if an ant were to grow by 1mm, the growth of the ant would be hugely significant. So when we compare things, just saying how different, in absolute terms, is not that helpful. And that is where percentage change comes in.

Consider the following two sentences:

- Felix, who is a human child of 2 years old, and quite small, grew 15cm in the past week.
- Mount Everest, which is a mountain, and very big, grew by 15cm in the past week.

Clearly the first sentence is false because children don't grow that much in a week. However the second sentence is quite plausible due to the summit being snowed upon. So 15cm can be a very large amount and also a very small amount, depending upon the context. Expressing the change as a percentage takes into account the context of the problem.

The formula we will use here should be familiar to you:

$$\text{Percentage} = \frac{\text{First quantity}}{\text{Second quantity}} \times 100$$

Example 14: Firstly, let's look at a simple example where we express one quantity as a percentage of another.

Question: Express 60p as a percentage of £40.

Method: Before you do anything else, make sure that the units are consistent. In this case they are not consistent so we should turn them both into the same units. In this case we could use pence or pounds. For this example, we will do the problem both ways, so you can see that we will get the same result either way. The important thing is consistency, not the units you choose.

Method 1

$$60\text{p} = £0.60$$

$$\text{Percentage} = \frac{0.6}{40} \times 100 = 1.5$$

Method 2

$$£40 = 4000\text{p}$$

$$\text{Percentage} = \frac{60}{4000} \times 100 = 1.5$$

Answer: 1.5%

Example 15: Now let's look at another.

Question: Express 75cm as a percentage of 2.5m

Method: $75\text{cm} = 0.75\text{m}$

$$\text{Percentage} = \frac{0.75}{2.5} \times 100 = 30$$

Answer: 30%

PERCENTAGE CHANGE – PROFIT & LOSS

As with differences in size, so increases and decreases in size must always be placed within the context of the relative size of the objects involved. This is where **percentage change** comes in very useful. There is a useful formula for calculating **percentage change** and here it is:

$$\text{Percentage change} = \frac{\text{change}}{\text{original value}} \times 100$$

Now the **change** can be calculated by finding the difference between the new and old values, i.e. subtracting one from the other. It is usual for the **change** to be a positive number in all circumstances. We deal with the fact that some are increases and some decreases by the language we use to describe them.

So **percentage profit** would be an increase and **percentage loss** would be a decrease.

Let us look at a few examples to see how this technique works.

Example 16: Firstly, let's look at a percentage profit example.

Question: Jabeer buys a car for £1500 and sells it for £1800.
What is Jabeer's percentage profit?

Method: The original price (£1500) will be the price paid for the car, as it is *what happened first*.
The sale price (£1800) will be the price the car was sold for, as it *what happened later*.

$$\text{Change} = \text{Final} - \text{Original} = 1800 - 1500 = 300$$

$$\text{Percentage profit} = \frac{300}{1500} \times 100 = 20$$

Answer: 20%

Example 17: Now let's look at a **loss** example.

Question: Richard bought a guitar for £560 and sold it several years later for £140.
What was his percentage loss?

Method: This time the **original** is £560, as *this happened first*.

$$\text{Loss} = 560 - 140 = 420$$

$$\text{Percentage loss} = \frac{420}{560} \times 100 = 75$$

Answer: 75%

EXERCISE 6

- Express the following as percentages. Round your answers appropriately, and as necessary.
 - £5 of £20
 - 6mm of 72mm
 - 18kg of 230kg
 - €12 of €98
 - 1 week of a year
 - 25mm of 4cm
 - 250g of 2kg
 - 25 minutes of 3 hours
 - 800seconds of 2.5 hours
- In a test George scored 11 marks out of a possible 25. What was his percentage score?
- Of the people in a room, 9 are men and 16 are women.
 - How many people are there in the room.
 - What percentage of the people in the room are men?
 - What percentage of the people in the room are women?
- Ellie bought bottles of fizzy pop to sell at a charity fete. She paid 20p per bottle to her supplier and sold each bottle for 30p. If she sold every bottle at the fete, what is her percentage profit?
- Alan bought a car for £12 000. One year later, he sold the car for £7680. What is his percentage loss?
- Charlie bought a box of 24 oranges for £12.00. She sold all of the oranges at 80p each. What was her percentage profit?
- Arlo works in a shop at a wage of £348.80 per week. After an increase in the minimum wage, Arlo's weekly wage is now £359.26. What was the percentage increase in the minimum wage?
- In a sale the price of a clarinet was reduced from £399 to £299. What was the percentage reduction in the price of the clarinet?

UNDOING A PERCENTAGE INCREASE OR DECREASE

Let us begin by examining a simple example.

Increase 100 by 10% and then decrease the result by 10%.

Increasing by 10% uses a multiplier of 1.1 and $100 \times 1.1 = 110$

Decreasing by 10% uses a multiplier of 0.9 and $110 \times 0.9 = 99$

This seems peculiar at first as you'd think that we should end up back at 100, but we don't. We have 'lost' 1. So what happened?

The answer is that we are finding 10% of different things. 10% of 100 is 10, but 10% of 110 is 11, hence the problem.

So if increasing and decreasing by a set percentage are not inverses of each other, what can we do?

As ever, in mathematics, the answer here lies in the question. When we increase 100 by 10%, we must multiply by 1.1, so in order to undo this increase, we must do the inverse operation, which is to divide by 1.1.

$$100 \times 1.1 = 110$$

$$110 \div 1.1 = 100$$

This works in exactly the same way with decreases. To decrease 100 by 10%, we multiply by 0.9.

To undo this calculation we will divide by 0.9.

$$100 \times 0.9 = 90$$

$$90 \div 0.9 = 100$$

Undoing a percentage increase or decrease will therefore use the same multiplier but we will divide rather than multiply.

Example 9: Mixed increase and decrease questions, introducing situations where we know the increased or decreased quantity and want to know the original quantity. This type of question is often referred to as **Reverse percentages**.

Question: VAT is a tax placed on most purchased goods. Currently this is set at 20% in the UK.

A company has an online shoe shop where the goods are priced excluding VAT.

- What would a customer pay, including the VAT, for goods priced at £85?
- If a customer has paid a total of £300 including the VAT, what was the price of the goods before the VAT was added on?

Method: a) Multiplier = $\frac{100 + p}{100} = \frac{100 + 20}{100} = 1.2$

$$85 \times 1.2 = 102$$

- b) The multiplier is the same as in part a) because the VAT is being added on. Because we are finding a price **before** the VAT is added, we will divide by the multiplier.

$$300 \div 1.2 = 250$$

Answer: a) £102
b) £250

Example 12: In this section we will examine examples where this reverse technique is very useful. We will use this whenever we want to find out **what the value was before...**

Question: Dan's has been awarded a 5% salary increase. His new salary is £12 915 per year. What was Dan's old salary?

Method: Following a timeline from past to present, the salary has been increased. This means that the multiplier is going to be an increase.

$$\text{Multiplier is } \frac{100 + p}{100} = \frac{100 + 5}{100} = 1.05$$

However we are looking back in time to the presale price so we are undoing a decrease, which means that we must divide by the multiplier.

$$\text{Previous salary} = 12\,915 \div 1.05 = 12\,300$$

Answer: Previous salary is £12 300

It is worth noticing that when you multiply 12 300 by the multiplier, you get

$$12300 \times 1.05 = \text{£}12\,915, \text{ which is his new salary.}$$

Example 13: This example uses a **decreasing** multiplier.

Question: A shirt is in a 25% off sale. It is priced at £36.00.
What was the pre-sale price?

Method: This is a sale so following a timeline from past to present, the price has been reduced. This means that the multiplier is going to be a decrease

$$\text{Multiplier is } \frac{100 - p}{100} = \frac{100 - 25}{100} = 0.75$$

However we are looking back in time to the presale price so we are undoing a decrease, which means that we must divide by the multiplier.

$$\text{Pre-sale price} = 36 \div 0.75 = 48$$

Answer: Pre-sale price is £48.00

It is worth noticing that when you multiply 48 by the multiplier, you get
 $48 \times 0.75 = 36$, which is our sale price.

EXERCISE 7

1. In a sale, televisions are reduced in price by 20%.
 - a) A television cost £325 before the sale, how much did it cost in the sale?
 - b) A television cost £300 in the sale, how much did it cost before the sale?
2.
 - a) On an island, the population of sheep decreased by 7% during 2016. There were 500 sheep at the beginning of 2016. How many sheep were there at the end of 2016?
 - b) On a nearby island, the same population decline of 7% was observed over the same time period. At the end of 2016 there were 279 sheep. How many were there at the beginning of 2016?
3. The manufacturer of a new fertilizer claims that vegetables will grow by as much as 20% each week.
 - a) A carrot had a mass of 0.4kg at the beginning of the week, what would its mass be at the end of the week, if the advertiser's claim are true?
 - b) At the end of the week using this new fertilizer, a marrow had a mass of 1.8kg. What would its mass have been at the beginning of the week, if the fertilizer works as advertised?
 - c) Harry decides to test the claims made by the fertilizer manufacturer. He weighs a sample of carrots and finds that their masses are on average 50g. After a week of using the fertilizer, he weighs another sample, and finds that they are now on average, 63g. What will Harry conclude about the new fertilizer?
4.
 - a) Jim sees a shirt in a sale. The sale banner says "24% off everything. The shirt was originally priced at £15.00. What is the sale price of the shirt?
 - b) Martha pays £12.58 for a skirt, in the sale. How much was the skirt when it was full price?
5.
 - a) George buys a car in a showroom. Between his first and second visit, the price of the car increased by 6%. The old price of the car was £15 000. What will the new price be?
 - b) What would the original price have been before the increase if the new price of the car is £18 020?
6.
 - a) I invest £3000 in a business. I am promised a 5% return from my investment over the first year. What will my investment be worth after one year?
 - b) A friend also invested in the same business on the same terms and after one year his investment is worth £4725. How much did my friend invest one year earlier?
7. A bottled spring water company increased the size of its bottles by 12% for a promotion.

- a) The company's biggest seller is the 300ml bottle. What is the new size (in mls) of the new bottle carrying the "12% extra free" label?
 - b) A larger bottle, which in the promotion, contains 560ml. What was the size (in mls) of the original, pre-promotion bottle?
8. A biscuit manufacturer holds a promotion where it increases the size of the packets of biscuits by 15% and labels the packets with "15% extra free".
 - a) What is the new mass of a packet which is usually 500g?
 - b) A packet in the promotion has a mass of 690g. What was the pre-promotion mass of this packet of biscuits?
9. A large department store has a promotion on boxed sets of cutlery (knives, forks, spoons etc.). The sign by the sale items reads 15% off everything.
 - a) The price of a set of cutlery (knives, forks, spoons etc.) is £399 before the sale. How much will it cost in the sale?
 - b) The price of another set is £350 in the sale. How much did it cost before the sale?
 - c) VAT (sales tax) is charged on all items at 20%. What would be the pre-sale, pre-VAT price of a set which was priced at £250 in the sale and including VAT?
10. Max receives a pay increase of 5%. Max now earns £231 per week. Work out Max's weekly wage before the increase.
11. In a sale, normal prices are reduced by 15%. The sale price of a CD player is £102. Work out the normal price of the CD player.
12. In a sale, all jackets are reduced by 24%. The sale price of a jacket is £120.00. Work out the pre-sale price of the jacket.
13. Employees in a firm receive a pay increase of 4%. After the increase, Mary earns £24 960 per year. How much did Mary earn before the increase.
14. VAT is added to the price of certain items at 22.5%. The price of one item, including VAT is £24.50.
 - a) What was the pre-tax price of the item?
 - b) How much VAT was charged on the item?

SUMMARY OF QUESTION TYPES

Heading	Result type	Formula	Keywords to look out for
Percentage of a quantity	Quantity	$q \times \frac{p}{100}$	of, percentage of, fraction of
Percentage increase	Quantity	$q \times \frac{100+p}{100}$	increased by, increase of, Tax, VAT
Percentage decrease	Quantity	$q \times \frac{100-p}{100}$	decreased by, less, discount, sale
Reverse percentages	Quantity	$q \div \frac{100+p}{100}$ (increase) or $q \div \frac{100-p}{100}$ (decrease)	before, last year, original, previously
One quantity as a percentage of another	Percentage	$\frac{1st\ quantity}{2nd\ quantity} \times 100$	express ... as a percentage of
Percentage change	Percentage	$\frac{change}{original} \times 100$	percentage profit, percentage loss
Compound change	Quantity	$q \times \left(\frac{100+p}{100}\right)^n$ (increase) $q \times \left(\frac{100-p}{100}\right)^n$ (decrease)	compound interest, principal, annual rate, depreciate, loss of ...per [time period]

- d) 6.72 e) 141.12 f) 5.28
3. £2.00
 4. 6 chocolates
 5. 408 songs
 6. £750
 7. 987 students
 8. 54 minutes
 9. 21 red sweets
 10. 600 boys
 11. 612 are not yellow
 12. 352 are likely to be right handed.
 13. Yes, he is correct. $a\%$ of $b = ab/100$ and $b\%$ of $a = ab/100$, which are the same.
 14. £3486
 15. $\frac{1}{200}$.
 16. 60
 17. 60
 18. £6.50
 19. 50
 20. £87.50
 21. 600
 22. 60
 23. 24 600

ANSWERS TO EXERCISE 3 - INCREASING A QUANTITY BY A PERCENTAGE

- | | | |
|-------------|--------------|-------------|
| 1. a) 1.4 | b) 1.53 | c) 1.06 |
| d) 1.175 | e) 3 | f) 4.5 |
| g) 2 | h) 1.01 | i) 1.003 |
| 2. a) 101 | b) 99 | c) 14.7 |
| d) 27 | e) 51.75 | f) 67.2 |
| g) 170.24 | h) 180 | i) 94.34 |
| 3. a) £6.60 | b) 6.5 kg | c) 26.1 cm |
| d) £76.70 | e) 44.1 l | f) £216 |
| g) 198.32 m | h) 340.48 cm | i) 116.6 mm |

4. £10 925
5. 4.2 kg
6. £73.60
7. 901 seals
8. £2700
9. £69.60
10. £223.60
11. £1.40
12. £1833.40
13. £36.56
14. 40 hours 15 minutes
15. 60mm

ANSWERS TO EXERCISE 4 - DECREASING A QUANTITY BY A PERCENTAGE

1.

a 0.9	b 0.8	c 0.99
d 0.66	e 0.72	f 0.58
g 0.94	h 0.92	i 0.975
2.

a 90	b 320	c 960
d 265.68	e 239.87	f 215.04
g 126.72	h 471.04	i 26.19
3.

a £16.80	b £262.20	c £694.41
d 681.74 cm	e 39.6 m	f 317.9 m
g 172.05 m	h 183.12 mm	i 25.68 cm
4. £11 000
5. 1148 mm
6. £47.60
7. 49.7 mm
8. 84 kg
9. £273.60 (can be rounded eg. £270 (2 s.f.))
10. 86 400 000
11.
 - a) £2 200
 - b) £2 464
 - c) Decreasing by a percentage and then increasing the result by the same percentage does not take you back to where you were at the start.
12. Gail was not correct. Increasing by 10% ($\times 1.1$) and then decreasing by 10% ($\times 0.9$) gives an overall reduction by 1%. ($1.1 \times 0.9 = 0.99$)

13. $0.86 \times 0.84 \times 0.85 = 0.61404$, which represents a 39.6% (3 s.f.) reduction overall.

ANSWERS TO EXERCISE 5 – REPEATED INCREASES AND DECREASES OVER TIME.

1. a) £364 b) 393.70 c) 766.89
2. a) 17.1kg b) 19.494 c) 27.534
3. a) 3.29m b) 12 years
4. a) 1557 seals b) 971 seals
5. £63.37
6. £808.87
7. £18 403.20
8. £1510.89

ANSWERS TO EXERCISE 6 – PERCENTAGE CHANGE.

1. a) 25% b) 8.3% c) 7.83%
d) 12.24% e) 1.92% f) 62.5%
g) 12.5% h) 13.89% i) 8.89%
2. 44
3. a) 25 b) 36% men c) 64% women
4. 50%
5. 36%
6. 60%
7. $2.9988\% = 3\%$
8. 25%

ANSWERS TO EXERCISE 7 - INCREASING AND DECREASING A QUANTITY BY A PERCENTAGE

1. a) £260
b) £375
2. a) 465 sheep
b) 300 sheep
3. a) 0.48kg
b) 1.5kg
c) The fertilizer has produced better than expected results. A 26% increase.
4. a) £11.10
b) £17
5. a) £15 900
b) £17 000
6. a) £3 150
b) £4 500

7. a) 336ml
b) 500ml
8. a) 575g
b) 600g
9. a) £339.15
b) £411.76
c) £245.10
10. £220
11. £120
12. £139.53
13. £24000
14. a) £20 b) £4.50